Problems PreCal 1508 Review, September 27, 2011
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1. Find the *slope-intercept form* of the equation of the line passing through the two points $(1, -2), (2, -3)$.

   Solution. The first step is to determine the slope of the line (or use the point-point formula). The slope is given by

   $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-2)}{2 - 1} = \frac{-1}{1} = -1$$

   Thus we have $y = -x + b$ and we know that it go through the point $(1, -2)$. So in order to find b, we can simply plug in the point and solve

   $$-2 = -(1) + b \Leftrightarrow -1 = b$$

   So the *slope-intercept form* of the equation is $y = -x - 1$ □

2. Find the *slope-intercept form* of the equation of a line passing through the point $(-9, 9)$ and perpendicular to line $y = 3x - 2$

3. Determine the implied domain for the following functions

   (a) $f(x) = x^3 + 2x + 9$

   Solution. The domain for all polynomials is known to be all real numbers, so the domain is $\mathbb{R}$ or alternatively $(-\infty, \infty)$. □

   (b) $h(x) = \frac{2x}{\sqrt{x-3}}$

   (c) $q(x) = \frac{9}{x^2-9}$

   Solution. We can see that $q(x)$ is a rational function, a polynomial divided by another polynomial. And we know that we cannot divide by zero, thus we must exclude from the domain whatever values of x make the denominator 0. The denominator is $x^2 - 9$, and so we set $0 = x^2 - 9$ and solve.

   $$0 = x^2 - 9 \Leftrightarrow 9 = x^2 \Leftrightarrow x = \pm 3$$

   So $-3, 3$ are not in the domain, and the domain can be written as $\mathbb{R} \setminus \{-3, 3\}$ or $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ □

4. For the following functions find $f \circ g$, $f - g$, $\frac{f}{g}$

(a) $f(x) = 3x - 9, \quad g(x) = x^2 + 7$

(b) $f(x) = \sqrt{x-1}, \quad g(x) = x^4 + 2$

Solution. i.

$$(f \circ g)(x) = f(g(x)) = \sqrt{(x^4+2) - 1} = \sqrt{x^4+1} \neq x^2 + 1$$

ii.

$$(f-g)(x) = f(x) - g(x) = \sqrt{x-1} - (x^4+2) = \sqrt{x-1} - x^4 - 2$$

iii.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x-1}}{x^4+2}$$

Note that since the denominator doesn't have any real zeros, that the domain of $\frac{f}{g}$ is all real numbers except those that would prevent you from taking the square root in the numerator. So the domain is all reals x, so that $x-1 \geq 0 \iff x \geq 1$. □

5. Find the vertex and the x-intercepts of the following quadratic functions

(a) $f(x) = x^2 - 9x + 2$

(b) $f(x) = 4x - 1 - 4x^2$

Solution. We can rewrite $f(x)$ as $f(x) = ax^2 + bx + c$, so it is $f(x) = -4x^2 + 4x - 1$. The vertex of a quadratic function is given by $(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)$. Since $a = -4$ and $b = 4$ we have...

$$-\frac{b}{2a} = -\frac{4}{2(-4)} = -\frac{1}{-2} = \frac{1}{2}$$

and

$$f\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 1 = -1 + 2 - 1 = 0$$

The vertex is on the $x-$axis, so we can deduce that it is the only x intercept, however you can check this with the discriminant, $b^2 - 4ac$

$$b^2 - 4ac = 4^2 - 4(-4)(-1) = 16 - 16 = 0$$

. So the quadratic formula is

$$x = \frac{-b \pm 0}{2a}$$

Which is the same as the x-coordinate of the vertex. □

6. Find the inverse function (if it exists) of the following functions

   (a) $f(x) = \frac{2}{\sqrt{x}}$

   Solution. Graphing will reveal that the function passes the horizontal line test, since the function is *one-to-one*. We can continue by trying to find the inverse function. First write $f(x)$ as $y = \frac{2}{\sqrt{x}}$ and then switch the two variables and solve for y...

$$x = \frac{2}{\sqrt{y}} \Longleftrightarrow \sqrt{y}x = 2 \Longleftrightarrow \sqrt{y} = \frac{2}{x} \Longleftrightarrow y = \frac{4}{x^2} = f^{-1}(x)$$

   We still must consider the domain of $f^{-1}(x)$. Which we know to be the range of $f(x)$. The range of $f(x)$ is all positive real numbers except 0, $(0, \infty)$ (as graphing would reveal), thus the domain of the inverse function is all positive real numbers except 0. □

   (b) $h(x) = (x - 2)^2 - 9$

   Solution. Squaring usually means there will be no inverse function, we can verify this by graphing or by noticing that for $x = -2$ and $x = 6$

$$h(-2) = ((-2) - 2)^2 - 9 = (-4)^2 - 9 = 16 - 9 = 7$$

$$h(6) = (6 - 2)^2 - 9 = 16 - 9 = 7$$

   Thus the function cannot pass the horizontal line test. □

7. Find all zeros of the functions (Real and Imaginary)

   (a) $f(x) = x^2 + 1$

   Solution. There are exactly 2 roots, since the degree of $f(x)$ is 2, so we apply the quadratic formula!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Longleftrightarrow x = \frac{\pm\sqrt{0 - 4}}{2} \Longleftrightarrow x = \frac{\pm 2\sqrt{-1}}{2} \Longleftrightarrow x = \pm i$$

   So the roots are $\pm i$ and we can write $f(x)$
   as $f(x) = (x - i)(x + i)$ □

   (b) $f(x) = x^2 + x + 2$

   (c) $f(x) = 2x^4 - 6x^2 - 4x^2 + 20x - 24, \quad \text{given } f(1 + i) = 0$

8. Write $f(x)$ as $f(x) = q(x)d(x) + r(x)$ (Remainder Theorem)

   (a) $f(x) = x^3 + 2x + 9, \quad d(x) = x - 3$

   Solution. Use Long or synthetic division to get the quotient and remainder
   $$f(x) = (x-3)(11 + 3x + x^2) + 42$$
   □

   (b) $f(x) = x^5 + 3x^4 + 9x + 2, \quad d(x) = x + 2$

9. Determine if -3 is a root of $f(x) = 2x^4 - 6x^3 - 38x^2 + 54x + 180$ (use synthetic division)

   Solution. It is. You can divide by 2 first to make your work easier. $f(x) = 2x^4 - 6x^3 - 38x^2 + 54x + 180 = 2(x-5)(x-3)(x+2)(x+3)$ □

10. Write the definitions of the following

    (a) The inverse function of f

    (b) A polynomial

    (c) Root of a function

    (d) Rational Function

(Select) Solutions to the problems will be available at
`http://rylai.dyndns.org/review_1.pdf`
You can reach me via e-mail at `awknaust@miners.utep.edu`
I expect you all to do well on the exam! Good Luck!