

Please do the problems that you feel will help your group the most first (you don't have to do them in order). All handouts are available at <http://alex.knaust.info/pltlfall2011/>

1. Solve this system of equations using the inverse matrix.

$$\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

2. Find the matrix  $A$  if

$$A = \begin{bmatrix} x & 2 \\ 6 & y \end{bmatrix} \quad A^2 = \begin{bmatrix} 13 & 8 \\ 24 & 37 \end{bmatrix}$$

3. Lets discover some useful properties of the determinant function of matrices

- (a) Find the determinant of  $A = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$

- (b) What is the determinant of  $A^{-1}$  and how is it related to the determinant of  $A$ ?

- (c) Show why  $\det(k \cdot M) = k \cdot \det(M)$  if  $M$  is a  $2 \times 2$  matrix and  $k$  is some real number?

- (d) Is  $\det(Q \cdot R) = \det(Q) \cdot \det(R)$  for two  $2 \times 2$  matrices  $Q$  and  $R$ ?

- (e) Is  $\det(Q + R) = \det(Q) + \det(R)$  ?

4. You have found at least two ways to solve systems of equations, one by performing row-operations on an augmented matrix, and another by multiplying by the inverse of the coefficient matrix. Why do they both work?

I have the augmented matrix  $A_0 = \left[ \begin{array}{cc|c} 1 & 2 & 0 \\ 3 & 4 & 0 \end{array} \right]$  and perform the Row-operation  $R_2 = -3R_1 + R_2$  to get the augmented matrix  $A_1 = \left[ \begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -2 & 0 \end{array} \right]$

- (a) How could I have written this equation as  $A \cdot X = B$ ? ( $X$  is a matrix of unknowns...)

- (b) How can I describe the row operation as multiplication with a matrix? (Which matrix  $C$  must I multiply by  $A$  to get to  $A_1$ ?)

- (c) Can all row operations be represented this way?

- (d) If they can, can you explain how using Gauß-Jordan elimination gives you the same result as multiplying by the inverse?