

Please do the problems that you feel will help your group the most first (you don't have to do them in order). All handouts are available at <http://alex.knaust.info/pltlfall2011/>

1. Show that the following is true by combining the fractions on the left hand side.

$$\frac{1}{x+1} + \frac{3}{x+2} - \frac{2}{(x+2)^2} = \frac{4x^2 + 11x + 8}{(x+1)(x+2)^2}$$

2. Write the form of the partial fraction decomposition for the following rational functions (do not solve for constants  $A, B, \dots$ )

$$\text{a) } \frac{x+4}{x^2(3x-1)^2} \quad \text{b) } \frac{x^2}{x^4-2x^2-8} \quad \text{c) } \frac{x^4+2x^3+4x^2+8x+2}{x^3+2x^2+x}$$

3. Write the partial fraction decomposition for the following rational functions (solve for constants)

$$\text{a) } \frac{3}{x^4+x} \quad \text{b) } \frac{x^2-4x+7}{(x+1)(x^2-2x+3)} \quad \text{c) } \frac{x^2-4x}{x^2+x+6}$$

4. Perform either Gaussian Elimination with back-substitution or Gauß-Jordan elimination.

$$\text{a) } \begin{cases} -x + y = 4 \\ 2x - 4y = -34 \end{cases} \quad \text{b) } \left[ \begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{array} \right] \quad \text{c) } \begin{cases} 3x + -2y + z = 15 \\ -x + y + 2z = -10 \\ x - y - 4z = 14 \end{cases}$$

5. Swapping rows in an augmented matrix will not alter the solutions. True or False?
6. Describe the difference between a matrix in row-echelon form and a matrix in reduced-row-echelon form, give examples of matrices in both forms.
7. A polynomial,  $f$ , of degree 3 goes through the following points

$$(-1, -5), (1, -1), (2, 1), (3, 11)$$

Find the function  $f(x)$ .

8. My peer leader told me that I can also do an analogue of partial fraction decomposition for rational numbers, using their prime factors as the denominators of the partial fractions, i.e.

$$\frac{1}{18} = \frac{1}{2 \cdot 3^2} = \frac{A}{2} + \frac{B}{3} + \frac{C}{3^2}$$

Here he meant a fraction to be proper if the numerator is less than the denominator.

- (a) What is a potential decomposition of  $\frac{4}{30}$
  - (b) Do you think he is right (Is this always possible)?
  - (c) Is the decomposition unique if there is one?
9. When is a polynomial irreducible? Can you apply the same concept to say when an integer is irreducible?